Letters

Corrections to "The Scattering Parameters and Directional Coupler Analysis of Characteristically Terminated Asymmetric Coupled Transmission Lines in an Inhomogeneous Medium"

Krzysztof Sachse

Haste in preparing the above paper[1] unfortunately entailed certain errors in equations (3), (6), (10a), (11), and (12). These consist in either a change of sign or index or omission of terms in the expressions. Equations (3), (6), (9), (10a), and (11) should be written as follows:

$$\delta_r(v_r) = (\beta_1+\beta_r)(\beta_{2(1)}-\beta_r)(\beta_2+\beta_r) - \beta_1\beta_2(\beta_{2(1)}+\beta_r)s^2 - \beta_1\beta_2(\beta_{2(1)}-\beta_r)d^2 \quad (3)$$

$$q = \frac{\sqrt{\beta_1\beta_2}\,s}{\beta_2+\beta_c} = \frac{\beta_2-\beta_\pi}{\sqrt{\beta_1\beta_2}\,s} \quad (6)$$

$$T_{23} = -T_{14}^* \quad (9)$$

$$S_{24} = S_{42} = S_{13}e^{j2\theta u} \quad (10a)$$

$$b_3 = 2\sqrt{\beta_1\beta_2}\,s\beta_2/b \qquad b_4 = 2\sqrt{\beta_1\beta_2}\,s\beta_1/b$$

$$a = (\beta_1+\beta_c)(\beta_2-\beta_c) - \beta_1\beta_2 s^2$$

$$b = (\beta_2+\beta_\pi)(\beta_1-\beta_\pi) - \beta_1\beta_2 s^2. \quad (11)$$

The pq^2 factor in the expressions (12) for the parameters $d_{21},\cdots,d_{34}$ should be replaced with $(pq)^2$. Moreover, the parameters d_{12}, d_{42}, and d_{13} in (12) should be written as follows:

$$d_{12} = (pq)^2\left(c_{12} + \frac{1}{q}c_{43} - qc_{42} - \frac{1}{q^2}c_{13}\right)e^{j\theta_c}(d/s)$$

$$d_{42} = (pq)^2\left(-q^2c_{12} - \frac{1}{q}c_{43} + c_{13} + qc_{42}\right)e^{-j\theta_\pi}(d/s)$$

$$d_{13} = (pq)^2\left(-\frac{1}{q}c_{12} - c_{43} + c_{42} + \frac{1}{q}c_{13}\right)e^{j\theta_c}(d/s). \quad (12)$$

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The author is with the Institute of Telecommunication and Acoustics, Wroclaw Technical University, Wyb. Wyspianskiego 27, 50-370 Wroclaw, Poland.
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[1] K. Sachse, *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 417–425, Apr. 1990.

Comments on "Formulas Useful for the Synthesis and Optimization of General, Uniform Contradirectional Couplers"

Krzysztof Sachse

In the above paper[1] explicit formulas that allow the evaluation and optimization of a very general class of contradirectional couplers are derived using a Y-parameter matrix given by Tripathi [1, eq. (22)]. An asymmetric, lossless, uniformly coupled line 4-port with line length l, lumped equalizing capacitors C_p, a source with admittance Y_G at port 1, and loading admittances Y_{1T} at port 4 and Y_{2T} at ports 2 and 3 is considered. Certain useful relations between the 4-port S parameters and these external loads assumed as real entities are determined in order to optimize conditions for contradirectional coupler operation. The optimization is carried out—using the proposed iterational procedure—by choosing C_p, Y_G, Y_{1T}, Y_{2T}, and length l in such a way that the coupler is perfectly matched and isolated at any chosen frequency. Specialized cases of a homogeneous coupler, a symmetric coupler, and couplers optimized with and without using capacitors are discussed, but there is nothing about the ideal asymmetric inhomogeneous coupled line coupler [2], [3] perfectly matched and isolated at all frequencies.

The S parameters of the couplers are expressed by admittances Y_G, Y_{1T}, and Y_{2T} and normal-mode parameters of asymmetric coupled lines (voltage mode numbers R_c and R_π and mode admittances Y_{c1} and $Y_{\pi 1}$). Some of the formulas contain square roots of the expressions $(-R_cR_\pi)$ and $Y_{c1}Y_{\pi 1}$, which, as noted by the author of the paper in question, can be negative. Those formulas can always be used because they are always combined in such a way that no complication arises. There are no references which could confirm that the mentioned case of unusual behavior of normal-mode parameters exists, and there is no analysis of the coupler which has $(-R_cR_\pi)<0$ together with $Y_{c1}Y_{\pi 1}<0$. Usually $(-R_cR_\pi)>0$ and mode admittances Y_{c1} and $Y_{\pi 1}$ are positive. In the literature there are many papers presenting the characteristics of normal-mode parameters for several structures of asymmetric coupled lines. Only in [2]–[4], to our knowledge, has this unusual behavior of coupled line parameters been numerically revealed. The first of them, [4], gave rise to considerable discussion and controversy.

Limiting the discussion to the cases of a homogeneous coupler, a symmetric coupler, and couplers with and without equalizing capacitors in the paper dealing with general, uniform contradirectional couplers makes it possible to restate the principles of the new coupler [2], [3] whose ideal properties do not

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The author is with the Institute of Telecommunication and Acoustics, Wroclaw Technical University, Wyb. Wyspianskiego 27, 50-370 Wroclaw, Poland.
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[1] F. Sellberg, *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 1000–1010, Aug. 1990.

impose the necessity of achieving equalization of the velocities of the two modes. The investigators frequently state that only if the velocities of two modes are equal can the contradirectional coupler be perfectly matched and isolated at all frequencies. It is worth noting that the two methods of analyzing the new coupler, namely the method applying the coupled-mode formulation [2] and [3] and the method proposed by Tripathi [1] and developed by Sellberg in the paper in question, used for calculations of the coupler responses lead to the same results.

It has been proved [2], [3] that the structure of inhomogeneous asymmetric coupled lines can comprise an ideal contradirectional coupler if the inductive k_L and capacitive k_C coupling coefficients of the coupled lines are equal and the terminating admittances Y_{iT} are equal to the characteristic admittances Y_i $(i=1,2)$ of uncoupled lines. The S parameters of this ideal coupler can be written in the following form (cf. [2, eqs. (1) and (2)] and [3, eqs. (10a) and (10b)]):

$$
\begin{aligned}
&S_{11}=S_{22}=S_{33}=S_{44}=0\\
&S_{13}=S_{31}=S_{24}=S_{42}=0\\
&S_{12}=S_{21}=S_{34}=S_{43}=\frac{jK\sin\theta}{\sqrt{1-K^2}\cos\theta+j\sin\theta}\\
&S_{14}=S_{41}=\frac{\sqrt{1-K^2}}{\sqrt{1-K^2}\cos\theta+j\sin\theta}e^{-j\theta u}\\
&S_{23}=S_{32}=S_{14}e^{j2\theta u}
\end{aligned}
\tag{1a}
$$

where

$$
K=\frac{2q}{1+q^2}\qquad q=\frac{\sqrt{\beta_1\beta_2 s}}{\beta_c+\beta_2}=\frac{\beta_2-\beta_\pi}{\sqrt{\beta_1\beta_2 s}}\qquad s=\frac{k_L+k_C}{2}
$$
$$
\theta=\frac{\theta_c+\theta_\pi}{2}\qquad u=\frac{\theta_c-\theta_\pi}{\theta_c+\theta_\pi}\qquad \theta_{c(\pi)}=\beta_{c(\pi)}l. \tag{1b}
$$

Here $\beta_{1(2)}$ and $\beta_{c(\pi)}$ are phase constants of uncoupled lines and propagation constants of the normal modes, respectively.
The following identity holds if $k_L=k_C$ (cf. [3, eq. (5b)]):

$$\beta_c-\beta_\pi=\pm(\beta_1-\beta_2). \tag{2}$$

This identity shows that the phase velocities of the normal modes, propagating in the structure under the condition that the coupling coefficients be equal, can be substantially different. Under this condition the normal mode parameters satisfy the following relations (cf. [2, eqs. (4) and (5)] and [3, eqs. (14) and (15)]):

$$Y_{c1}=\pm Y_1\qquad Y_{\pi 1}=\mp Y_1 \tag{3a}$$

and

$$R_cR_\pi=Y_1/Y_2. \tag{3b}$$

Moreover, the effective coupling coefficient, K, from (1a) can be expressed by the voltage mode numbers (cf. [2, eq. (6)] and [3, eq. (17)]):

$$K=\frac{2\sqrt{R_cR_\pi}}{R_c+R_\pi}. \tag{4}$$

It can be easily proved, submitting eqs. (3) into equation (12) in the paper in question, that both the real and the imaginary part of the denominator of the expression S_{21}/S_{31} are equal to zero and the numerator of S_{21}/S_{31} is not equal to zero. The directivity of the coupler is infinite. This result confirms the ideal operation of the coupler, in which two normal modes propagate with different phase velocities, and negates the statement by Sellberg that perfect isolation for all frequencies can be achieved only in the case of homogeneous couplers. One can develop the synthesis and optimization for the two asymmetric couplers: the first, with equalizing capacitors C_p, which can perfectly match and isolate ports of the coupler at any chosen frequency, and the second, with multilayered structures of coupled lines, for which the equalization of the coupling coefficients can give perfect matching and isolation at all frequencies.

The coupler balance can be determined with the help of (1) and (4) as

$$S_{21}/S_{41}=j\frac{\sqrt{R_cR_\pi}}{R_c-R_\pi}\{\sin\theta_c+\sin\theta_\pi-j(\cos\theta_c-\cos\theta_\pi)\}. \tag{5a}$$

For $\theta_c\approx\theta_\pi$,

$$S_{21}/S_{41}\approx j\frac{\sqrt{R_cR_\pi}}{R_c-R_\pi}(\sin\theta_c+\sin\theta_\pi). \tag{5b}$$

Notice that formula (5b) is identical with (8) in Sellberg's paper, after introducing there relation (3a).

In the second part of his paper, Sellberg calculates characteristics of several compensated and uncompensated microstrip couplers. Three of them, namely three-strip Lange 3 dB and 10 dB couplers and an asymmetric 10 dB coupler ($Y_{1T}/Y_{2T}=2$), could be optimized in the way proposed in [2] and [3].

Reply[2] by Florian Sellberg[3]

In his comments on my paper, Sachse addresses the case of the ideal, asymmetric, inhomogeneous coupled-line coupler [2], [3], which is perfectly matched and isolated at all frequencies (if dispersion can be neglected). It is true that this rather extraordinary case was not given due tribute in my paper, although the possibility—and tractability—of the case with $Y_{c1}Y_{\pi 1}<0$ and $R_cR_\pi>0$ was mentioned. I regret that the hasty statement to the effect that a perfect isolation without compensating susceptance is possible only for a homogeneous coupler did find its way into the text. In fact, even in the case of a symmetric coupler, where $\beta_c=\beta_\pi$ can be reached in microstrip with a suitable overlay or with the right proportion of suspended substrate, the coupler is not strictly what one would label homogeneous. It is better described as compensated in a distributed and therefore frequency-insensitive manner.

The treatment in my paper could have stressed the general applicability more by everywhere replacing ωC_p with a susceptance B_p. A negative value of B_p covers the possibility of inductive compensation, which is only very casually mentioned under special case B. It is seen that the asymmetric case with equal inductive and capacitive coupling coefficients [2], [3], i.e., $R_cR_\pi=Y_1/Y_2$, $Y_{c1}=-Y_{\pi 1}=\pm Y_1$, and $Y_G=Y_1$, may be derived directly from (3) and (4) in my paper, as the condition

[2]Manuscript received December 31, 1990.
[3]The author is with the Swedish Institute of Microelectronics, IM, P.O. Box 1084, S-164 21 Kista, Sweden.
IEEE Log Number 9144297.

$\omega C_p = 0$ yields

$$[\cos(\beta_\pi l) - \cos(\beta_c l)] \cdot [Y_2 - Y_1/(R_c R_\pi)] = 0$$

$$Y_1 Y_2 = -\frac{Y_{c1} Y_{\pi 1}}{R_c R_\pi} \cdot \frac{Y_{\pi 1}\sin(\beta_\pi l) - Y_{c1}\sin(\beta_c l)}{Y_{\pi 1}\sin(\beta_c l) - Y_{c1}\sin(\beta_\pi l)}$$

$$Y_1^2 = -Y_{c1} Y_{\pi 1} \cdot \frac{R_c Y_{\pi 1}\cot(\beta_c l) - R_\pi Y_{c1}\cot(\beta_\pi l)}{R_\pi Y_{\pi 1}\cot(\beta_\pi l) - R_c Y_{c1}\cot(\beta_c l)}$$

Apart from the homogeneous—or possibly compensated—case, $\beta_c = \beta_\pi$, there also exists a third solution with the condition $\omega C_p = 0$, namely,

$$Y_1 / Y_2 = R_c R_\pi$$

$$\cos(\beta_\pi l)/\cos(\beta_c l) = R_c / R_\pi$$

$$Y_1^2 = -Y_{c1} Y_{\pi 1} \cdot \frac{Y_{\pi 1}\sin(\beta_\pi l) - Y_{c1}\sin(\beta_c l)}{Y_{\pi 1}\sin(\beta_c l) - Y_{c1}\sin(\beta_\pi l)}.$$

This solution is of the case 2 type in my paper and is narrowband. Thus it does not yield $S_{11} = 0$ for $Y_G = Y_1$ but rather

$$Y_G = Y_2 \cdot (Y_1 / Y_2)_{\text{broad-band}}.$$

When $Y_{c1} = -Y_{\pi 1}$, this narrow-band solution coincides with the broad-band solution above.

To optimize couplers in the neighborhood of the special ideal point $Y_{c1} = -Y_{\pi 1}$ without resorting to compensating susceptances the procedure in my paper, special case C, can be applied. With design for broad-band optimum, we may—besides equation (16c) in my paper for S_{11}—derive the following general expressions from (12c) and (13c):

$$S_{21}/S_{31} = -\frac{1}{(R_c - R_\pi)^2 a_7^2 \sin\vartheta_0} \cdot \left\{ \frac{(Y_{\pi 1} - Y_{c1})}{(Y_{\pi 1} + Y_{c1})} - a_6 \sin^2\vartheta_0 \right\}$$

$$S_{21}/S_{41} = j \cdot \sqrt{\frac{-R_c R_\pi}{Y_{c1} Y_{\pi 1}}} \cdot \frac{(Y_{\pi 1} - Y_{c1})}{(R_c - R_\pi)} \cdot [\cos\vartheta_0 - j \cdot a_7 (R_c + R_\pi)\sin\vartheta_0].$$

As $a_6 = 0$ and $a_7 = 1/(R_c + R_\pi)$ at the special point $Y_{c1} = -Y_{\pi 1}$, which is easily derived from my paper, we find, for the ideal coupler at the frequency f_0,

$$S_{11} = 0$$

$$S_{21}/S_{31} = \infty$$

$$S_{21}/S_{41} = \frac{2 \cdot \surd(R_c R_\pi)}{(R_c - R_\pi)} \cdot \exp[j \cdot (\pi/2 - \vartheta_0)]$$

in complete agreement with Sachse's comments above. Observe that my definition of c and π modes is chosen such that $R_c > R_\pi$! A consequence of this choice is also that only the lower sign in Sachse's relation (3a) above becomes physically realizable.

In spite of the wildly varying values of $R_{c,\pi}$ and $Y_{c,\pi}$ around the special point $Y_{c1} = -Y_{\pi 1}$, one finds a quite smooth variation of all externally observable parameters for the optimized coupler when the composition of the dielectric layers is changed.

References

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[2] K. Sachse, "Analysis of asymmetrical inhomogeneous coupled-line directional coupler," in *Proc. 18th European Microwave Conf.* (Stockholm), Sept. 12–16, 1988, pp. 985–990.

[3] K. Sachse, "The scattering parameters and directional coupler analysis of characteristically terminated asymmetric coupled transmission lines in an inhomogeneous medium," *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 417–425, Apr. 1990.

[4] K. Sachse and A. Sawicki, "Characteristics of asymmetric tightly coupled transmission lines in an inhomogeneous medium," in *Proc. 7th National Microwave Conf. MIKON-86* (Zakopane, Poland), Apr. 1986, part 1, pp. 233–236 (in Polish).